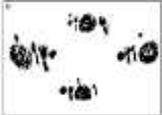
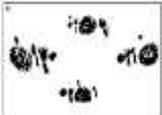


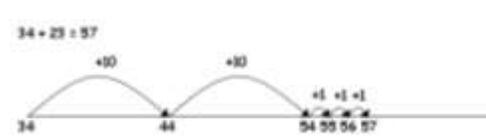
Progression In Calculations: Bilsdale Midcable Chop Gate and Carlton and Faceby CE Confederated Schools.

	Addition	Subtraction	Multiplication	Division
Stage 1	<p>Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, etc.</p>  <p>Bead strings or bead bars can be used to illustrate addition</p>  <p>8+2=10</p> <p>They use numberlines and practical resources to support calculation and teachers <i>demonstrate</i> the use of the numberline.</p>	<p>Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures etc.</p>  <p>Bead strings or bead bars can be used to illustrate subtraction including bridging through ten by counting back 3 then counting back 2.</p>  <p>6-2=4</p> <p>They use numberlines and practical resources to support calculation. Teachers <i>demonstrate</i> the use of the numberline.</p>	<p>Children will experience equal groups of objects.</p> <p>They will count in 2s and 10s and begin to count in 5s.</p> <p>They will work on practical problem solving activities involving equal sets or groups.</p> 	<p>Children will understand equal groups and share items out in and problem solving. They will count in 2s and 10s and later</p> 
Stage 2	<p>using pictures</p>  <p>Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3.</p>  <p>They use numberlines and practical resources to support calculation and teachers <i>demonstrate</i> the use of the numberline.</p> <p>Children then begin to use numbered lines to support their own calculations using a numbered line to count on in ones.</p> <p align="center">↓</p> <p>Partitioning Tens and Units eg 14 = 10 + 4</p>	<p>using pictures</p>  <p>Bead strings or bead bars can be used to illustrate subtraction including bridging through ten by counting back 3 then counting back 2.</p>  <p>13-5=8</p> <p>Children then begin to use numbered lines to support their own calculations, - counting back / counting on, depending on the size of the numbers subtracting.</p> <p>The numberline should also be used to show that 6 - 3 means the 'difference between 6 and 3' or 'the difference between 3 and 6' and how many jumps they are apart.</p>	<p>Children will experience equal groups of objects.</p> <p>They will count in 2s and 10s and begin to count in 5s.</p> <p>They will work on practical problem solving activities involving equal sets or groups.</p>  <p align="center">↓</p> <p>✓ Arrays Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.</p> 	<p>Children will understand equal groups and share items out in and problem solving. They will count in 2s and 10s and later</p> 

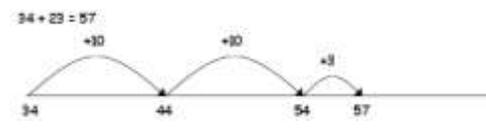
Progression In Calculations: Bilsdale Midcable Chop Gate and Carlton and Faceby CE Confederated Schools.

Stage 3

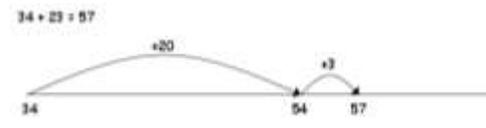
Children will begin to use 'empty number lines' themselves starting with the larger number and counting on.



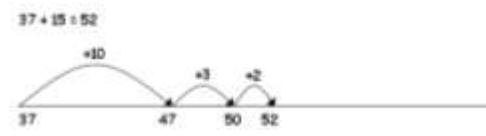
✓ Then helping children to become more efficient by adding the units in one jump (by using the known fact $4 + 3 = 7$).



✓ Followed by adding the tens in one jump and the units in one jump.

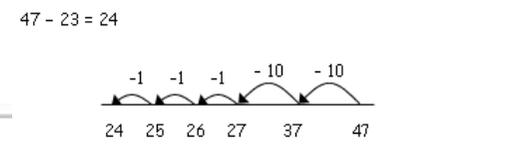


✓ Bridging through ten can help children become more efficient.

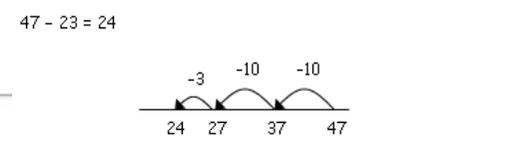


Children will begin to use empty number lines to support calculations.

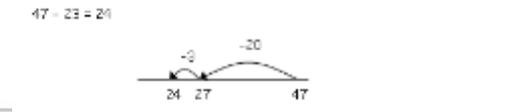
✓ **Either Counting back:**
First counting back in tens and ones.



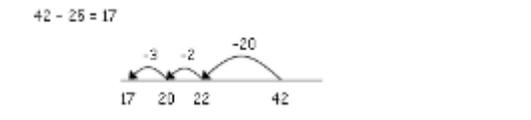
✓ Then helping children to become more efficient by subtracting the units in one jump (by using the known fact $7 - 3 = 4$).



✓ Subtracting the tens in one jump and the units in one jump.



✓ Bridging through ten can help children become more efficient.



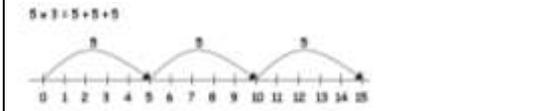
Or **Counting on:** Start from the number being taken away, bridge through a ten and then count in tens / count the tens in one jump to reach the number you are taking away from.

Know multiplication facts and can work out division facts for 2x, 5x and 10x tables.

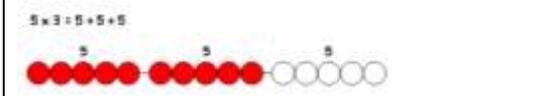
Children will develop their understanding of multiplication and use jottings to support calculation:

✓ **Repeated addition**
3 times 5 is $5 + 5 + 5 = 15$ or 3 lots of 5 or 5×3

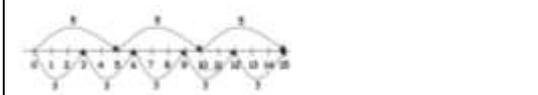
Repeated addition can be shown easily on a number line:



and on a bead bar:



✓ **Commutativity**
Children should know that 3×5 has the same answer as 5×3 . This can also be shown on the number line.



Children will develop their understanding of division and use jottings to support calculation

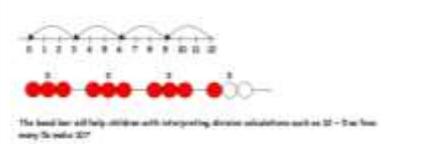
✓ **Sharing equally**
6 sweets shared between 2 people, how many do they each get?



✓ **Grouping or repeated subtraction**
There are 6 sweets, how many people can have 2 sweets each?



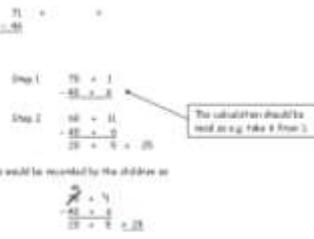
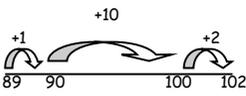
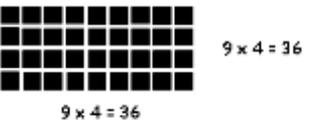
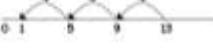
✓ **Repeated subtraction using a number line or bead bar**
 $12 \div 3 = 4$



✓ **Using symbols to stand for unknown numbers to complete equations using inverse operations**

$\square \div 2 = 4$ $20 \div \triangle = 4$ $\square \div \triangle = 4$

Progression In Calculations: Bilsdale Midcable Chop Gate and Carlton and Faceby CE Confederated Schools.

	Addition	Subtraction	Multiplication	Division
Stage 4	<p>Children will continue to use empty number lines with increasingly large numbers, including compensation where appropriate.</p> <p>✓ Count on from the largest number irrespective of the order of the calculation.</p>  <p>✓ Compensation</p>  <p style="text-align: center;">↓</p> <p>Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies.</p> <p>Expanded Method (Column Addition).</p> <p>Adding the least significant digits first.</p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> $\begin{array}{r} 67 \\ + 24 \\ \hline 11 \text{ (} 7 + 4 \text{)} \\ \underline{80} \text{ (} 60 + 20 \text{)} \\ 91 \end{array}$ </div> <div style="text-align: center;"> $\begin{array}{r} 267 \\ + 89 \\ \hline 12 \text{ (} 7 + 5 \text{)} \\ 140 \text{ (} 60 + 80 \text{)} \\ \underline{200} \\ 352 \end{array}$ </div> </div> <p style="text-align: center;">↓</p> <p>Column Addition (no brackets).</p> $\begin{array}{r} 67 \\ + 24 \\ \hline 11 \\ \underline{80} \\ 91 \end{array}$	<p>Children will continue to use empty number lines with increasingly large numbers.</p> <p>Children will begin to use informal pencil and paper methods (jottings).</p> <p>✓ Partitioning and decomposition</p> <ul style="list-style-type: none"> Partitioning - demonstrated using arrow cards Decomposition - base 10 materials <p>NOTE: When solving the calculation $89 - 57$, children should know that 57 does NOT EXIST AS AN AMOUNT it is what you are subtracting from the other number. Therefore, when using base 10 materials, children would need to count out only the 89.</p> $\begin{array}{r} 89 = 80 + 9 \\ - 57 \\ \hline 30 + 2 = 32 \end{array}$ <p>✓ Begin to exchange.</p>  <p style="font-size: small;">This would be recorded by the children as</p> $\begin{array}{r} 71 \\ - 45 \\ \hline 26 \end{array}$ <p>Where the numbers involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used.</p> <p>$102 - 89 = 13$</p> 	<p>Know multiplication and division facts for 2, 3, 4, 5, 6 and 10x tables.</p> <p>Children will continue to use:</p> <p>✓ Repeated addition</p> <p>4 times 6 is $6 + 6 + 6 + 6 = 24$ or 4 lots of 6 or 6×4</p> <p>Children should use number lines or bead bars to support their understanding.</p>   <p>✓ Arrays</p> <p>Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.</p>  <p>✓ Scaling</p> <p>e.g. Find a ribbon that is 4 times as long as the blue ribbon</p>  <p>✓ Using symbols to stand for unknown numbers to complete equations using inverse operations</p> <p>$\square \times 5 = 20$ $3 \times \triangle = 18$ $\square \times \circ = 32$</p> <p>✓ Partitioning</p> $38 \times 5 = (30 \times 5) + (8 \times 5) \\ = 150 + 40 \\ = 190$	<p>Ensure that the emphasis at this stage is on grouping rather than sharing.</p> <p>Children will continue to use:</p> <p>✓ An empty number line (either counting on / back in groups of).</p> <p style="font-size: small;">Children will use an empty number line to support their calculation.</p>  <p>Children should also move onto calculations involving remainders.</p>  <p>✓ Using symbols to stand for unknown numbers to complete equations using inverse operations</p> <p>$26 \div 2 = \square$ $24 \div \triangle = 12$ $\square \div 10 =$</p>

Progression In Calculations: Bilsdale Midcable Chop Gate and Carlton and Faceby CE Confederated Schools.

Stage 5

✓ Compact Method (Carry below the line).

$$\begin{array}{r} 625 \\ + 48 \\ \hline 673 \end{array} \qquad \begin{array}{r} 783 \\ + 42 \\ \hline 825 \end{array} \qquad \begin{array}{r} 347 \\ + 85 \\ \hline 432 \end{array}$$

Using similar methods, children will:

- ✓ add several numbers with different numbers of digits;
- ✓ begin to add two or more three-digit sums of money, with or without adjustment from the pence to the pounds;
- ✓ know that the decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. £3.59 + 78p.

✓ Expanded Method (Column Subtraction).

$$\begin{array}{r} 1 \\ 87 - \\ \hline 59 \\ \hline 8 \quad (17 - 9) \\ 20 \quad (70 - 50) \\ \hline 28 \end{array}$$



Compact Method

$$\begin{array}{r} 87 \\ - 59 \\ \hline 28 \end{array}$$

Children should:

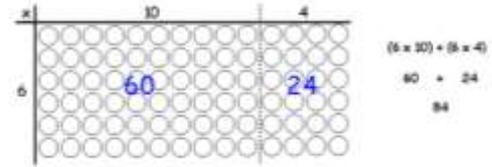
- ✓ be able to subtract numbers with different numbers of digits;
- ✓ using this method, children should also begin to find the difference between two three-digit sums of money, with or without 'adjustment' from the pence to the pounds;
- ✓ know that decimal points should line up under each other.

$$\begin{array}{r} 11.95 \\ - 3.58 \\ \hline 8.37 \end{array} \qquad \begin{array}{r} 1.50 \\ - 0.12 \\ \hline 1.38 \end{array}$$

Adjust from 7 to 6

Know all multiplication and division facts up to 10 x 10.

Children will continue to use arrays where appropriate leading into the Grid Method of multiplication.



✓ Grid method

TU x U

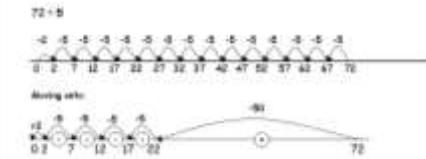
(Short multiplication - multiplication by a single digit)

$$23 \times 8$$

Children will approximate first
23 x 8 is approximately 25 x 8 = 200

$$\begin{array}{r} \times 20 \quad 3 \\ 8 \quad \boxed{160} \quad 24 \\ \hline 160 \\ + 24 \\ \hline 184 \end{array}$$

Children will develop their use of an empty number line to be able to subtract or count on in multiples of the divisor. Initially, should be multiples of 10s, 5s, 2s and 1s - numbers with which children are more familiar.



Then onto the vertical method: Chunking:

Short division TU ÷ U

$$\begin{array}{r} 23 \overline{) 184} \\ \underline{- 46} \\ 138 \\ \underline{- 104} \\ 34 \\ \underline{- 32} \\ 2 \end{array}$$

Leading to subtraction of other multiples.

$$\begin{array}{r} 26 \overline{) 96} \\ \underline{- 52} \\ 44 \\ \underline{- 38} \\ 6 \end{array}$$

Any remainders should be shown as integers, i.e. 14 remainder 14 r 2.

Children need to be able to decide what to do after division round up or down accordingly. They should make sensible decisions about rounding up or down after division.

Progression In Calculations: Bilsdale Midcable Chop Gate and Carlton and Faceby CE Confederated Schools.

	Addition	Subtraction	Multiplication	Division
<p>Stage 6</p>	<p>Children should extend the carrying method to numbers with at least four digits.</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> $\begin{array}{r} 587 \\ + 475 \\ \hline 1062 \\ \hline 1 \end{array}$ </div> <div style="text-align: center;"> $\begin{array}{r} 3587 \\ + 675 \\ \hline 4262 \\ \hline 1 \end{array}$ </div> </div> <p>Using similar methods, children will:</p> <ul style="list-style-type: none"> ✓ add several numbers with different numbers of digits; ✓ begin to add two or more decimal fractions with up to three digits and the same number of decimal places; ✓ know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. 3.2 m - 280 cm. 	<p>Compact Method: Decomposition</p> $\begin{array}{r} 644 \\ 744 \\ - 286 \\ \hline 458 \end{array}$ <p>Children should:</p> <ul style="list-style-type: none"> ✓ be able to subtract numbers with different numbers of digits; ✓ begin to find the difference between two decimal fractions with up to three digits and the same number of decimal places; <p>know that decimal points should line up under each other</p> <p>Where the numbers involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line can be used.</p> <div style="text-align: center;"> $1209 - 888 = 321$ </div>	<p>Quick recall of all multiplication and division facts up to 10 x 10.</p> <p>Grid method HTU x U (Short multiplication - multiplication by a single digit) 346 x 9 Children will approximate first 346 x 9 is approximately 350 x 10 = 3500</p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> $\begin{array}{r} \times \quad 300 \quad 40 \quad 6 \\ 9 \quad \boxed{2700} \quad \boxed{360} \quad \boxed{54} \end{array}$ </div> <div style="text-align: center;"> $\begin{array}{r} 2700 \\ + 360 \\ + 54 \\ \hline 3114 \end{array}$ </div> </div> <p>TU x TU (Long multiplication - multiplication by more than a single digit) 72 x 38 Children will approximate first 72 x 38 is approximately 70 x 40 = 2800</p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> $\begin{array}{r} \times \quad 70 \quad 2 \\ 30 \quad \boxed{2100} \quad \boxed{60} \\ 8 \quad \boxed{560} \quad \boxed{36} \end{array}$ </div> <div style="text-align: center;"> $\begin{array}{r} 2100 \\ + 560 \\ + 60 \\ + 36 \\ \hline 2736 \end{array}$ </div> </div> <p>Using similar methods, they will be able to multiply decimals with one decimal place by a single digit number, approximating first. They should know that the decimal points line up under each other. e.g. 4.9 x 3 Children will approximate first 4.9 x 3 is approximately 5 x 3 = 15</p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> $\begin{array}{r} \times \quad 4 \quad 09 \\ 3 \quad \boxed{12} \quad \boxed{27} \end{array}$ </div> <div style="text-align: center;"> $\begin{array}{r} 12 \\ + 27 \\ \hline 147 \end{array}$ </div> </div>	<p>Children will continue to use written methods to solve short division TU ÷ U.</p> <p>Children can start to subtract larger multiples of the divisor, e.g. 30x</p> <p>Chunking: HTU ÷ U</p> <div style="text-align: center;"> $196 \div 6$ </div> <p>Any remainders should be shown as integers, i.e. 14 remainder 2 or 14 r 2.</p> <p>Children need to be able to decide what to do after division and round up or down accordingly. They should make sensible decisions about rounding up or down after division.</p>

Progression In Calculations: Bilsdale Midcable Chop Gate and Carlton and Faceby CE Confederated Schools.

	Addition	Subtraction	Multiplication	Division																														
Stage 7	<p>Children should extend the carrying method to number with any number of digits.</p> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="text-align: center;"> $\begin{array}{r} 7488 \\ + 1485 \\ \hline 8973 \\ \hline 111 \end{array}$ </div> <div style="text-align: center;"> $\begin{array}{r} 4584 \\ + 5948 \\ \hline 10532 \\ \hline 111 \end{array}$ </div> <div style="text-align: center;"> $\begin{array}{r} 42 \\ 6472 \\ 786 \\ 3 \\ + 4681 \\ \hline 111 \end{array}$ </div> </div> <p style="margin-top: 10px;"><i>Using similar methods, children will</i></p> <ul style="list-style-type: none"> ✓ add several numbers with different numbers of digits; ✓ begin to add two or more decimal fractions with up to four digits and either one or two decimal places; ✓ know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. $401.2 + 26.85 + 0.71$. 	<p>Decomposition</p> <div style="margin-top: 10px;"> $\begin{array}{r} 3\ 13\ 1 \\ 6467 \\ - 2684 \\ \hline 3783 \end{array}$ </div> <p style="margin-top: 10px;"><i>Children should:</i></p> <ul style="list-style-type: none"> ✓ be able to subtract numbers with different numbers of digits; ✓ be able to subtract two or more decimal fractions with up to three digits and either one or two decimal places; ✓ know that decimal points should line up under each other. <p style="margin-top: 10px;">Where the numbers are involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used.</p> <div style="margin-top: 10px;"> $3882 - 2987 = 895$ </div>	<p>ThHTU x U (Short multiplication - multiplication by a single digit) 4346×8</p> <p>Children will approximate first 4346×8 is approximately $4346 \times 10 = 43460$</p> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="text-align: center;"> <table border="1" style="border-collapse: collapse;"> <tr><td>x</td><td>4000</td><td>300</td><td>40</td><td>6</td></tr> <tr><td>8</td><td>32000</td><td>2400</td><td>320</td><td>48</td></tr> </table> </div> <div style="text-align: center;"> $\begin{array}{r} 32000 \\ + 2400 \\ + 320 \\ + 48 \\ \hline 34768 \end{array}$ </div> </div> <p style="margin-top: 10px;">HTU x TU (Long multiplication - multiplication by more than a single digit) 372×24</p> <p>Children will approximate first 372×24 is approximately $400 \times 25 = 10000$</p> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="text-align: center;"> <table border="1" style="border-collapse: collapse;"> <tr><td>x</td><td>300</td><td>70</td><td>2</td></tr> <tr><td>20</td><td>6000</td><td>1400</td><td>40</td></tr> <tr><td>4</td><td>1200</td><td>280</td><td>8</td></tr> </table> </div> <div style="text-align: center;"> $\begin{array}{r} 6000 \\ + 1400 \\ + 1200 \\ + 280 \\ + 40 \\ + 8 \\ \hline 8928 \end{array}$ </div> </div> <p style="margin-top: 10px;"><i>Using similar methods, they will be able to multiply decimals with up to two decimal places by a single digit number and then two digit numbers, approximating first. They should know that the decimal points line up under each other.</i></p> <p><i>For example:</i> 4.92×3</p> <p>Children will approximate first 4.92×3 is approximately $5 \times 3 = 15$</p> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="text-align: center;"> <table border="1" style="border-collapse: collapse;"> <tr><td>x</td><td>4</td><td>99</td><td>002</td></tr> <tr><td>3</td><td>12</td><td>27</td><td>006</td></tr> </table> </div> <div style="text-align: center;"> $\begin{array}{r} 12 \\ + 07 \\ + 006 \\ \hline 14.76 \end{array}$ </div> </div>	x	4000	300	40	6	8	32000	2400	320	48	x	300	70	2	20	6000	1400	40	4	1200	280	8	x	4	99	002	3	12	27	006	<p>Children will continue to use written methods to solve short division $TU \div U$ and $HTU \div U$.</p> <p>Chunking: HTU \div TU</p> <div style="margin-top: 10px;"> $972 \div 24$ </div> <p style="margin-top: 10px;">Any remainders should be shown as fractions, i.e. if the children were dividing 32 by 10, the answer should be shown as $3 \frac{2}{10}$ which could then be written as $3 \frac{1}{5}$ in its lowest terms.</p> <p style="margin-top: 10px;">Extend to decimals with up to two decimal places. Children should know that decimal points line up under each other.</p> <div style="margin-top: 10px;"> $275 \div 7$ </div>
x	4000	300	40	6																														
8	32000	2400	320	48																														
x	300	70	2																															
20	6000	1400	40																															
4	1200	280	8																															
x	4	99	002																															
3	12	27	006																															

By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

Children should not be made to go onto the next stage if:
they are not ready.
they are not confident.

Children should be encouraged to approximate their answers before calculating.
Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.